

Packet 1 for Unit 2 Intercept Form of a Quadratic Function

M2 Alg 2

Assignment 2A: Graphs of Quadratic Functions in Intercept Form (Section 4.2)

In this lesson, you will:

- Determine whether a function is linear or quadratic.
- Locate the vertex, axis of symmetry, and intercepts of the graph of a quadratic function given in intercept form.
- Convert a quadratic function from intercept form to standard form.

Part 1: Linear or Quadratic?

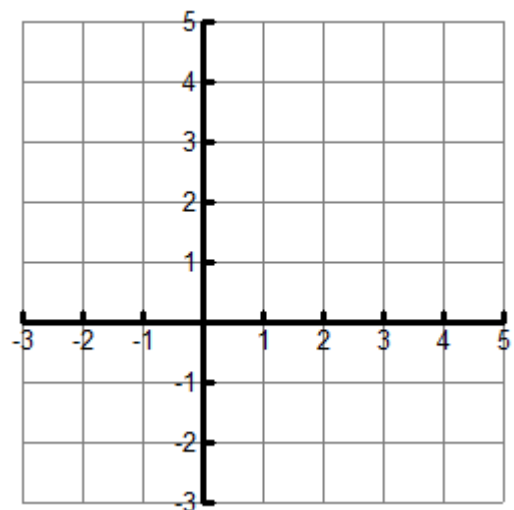
1. Any quadratic function can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$. What kind of equation would it be if $a = 0$?
2. Identify values of a , b , and c , and tell whether the equation is linear or quadratic.

| Equation | a | b | c | Linear or Quadratic? |
|--------------------|-----|-----|-----|----------------------|
| $y = x^2 + 4x + 7$ | | | | |
| $y = 3x + 5$ | | | | |
| $y = x^2 + 4$ | | | | |
| $y = x^2 - 7x$ | | | | |
| $y = 10$ | | | | |
| $y = 3x^2$ | | | | |

Part 2: Key Features of a Quadratic Function's Graph

3. Consider the quadratic function $y = x^2 - 4x + 3$.
 - a. Use the equation to complete the table. Then graph the points and connect with a smooth curve to complete the parabola.

| x | y |
|-----|-----|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |



- b. Give the coordinates of the...

vertex:

y-intercept:

x-intercept(s):

Part 3: Standard Form and Intercept Form

The equation $y = x^2 - 4x + 3$ is written in standard form.

4. $y = (x-1)(x-3)$ describes the same graph but is written in intercept form.
- How are the numbers in the equation related to the x-intercepts in #3b on page 2?
 - Simplify the right side of $y = (x-1)(x-3)$ to show that it is the same as $y = x^2 - 4x + 3$.

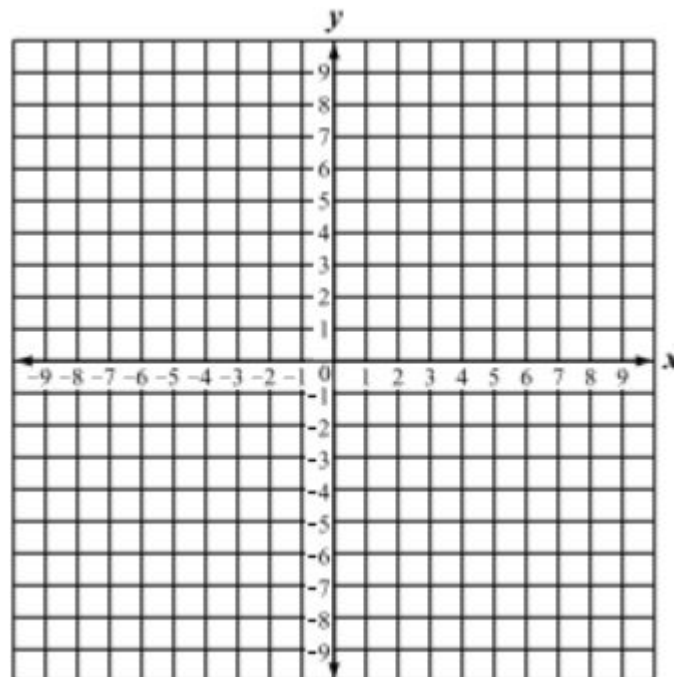
Part 4: Finding Key Features of a Quadratic Graph in Intercept Form

Consider the equation $y = (x-2)(x+4)$.

5. This equation is in intercept form because it follows the pattern $y = a(x-p)(x-q)$, where p and q are the x-intercepts.
- Find the values of a , p , and q in $y = (x-2)(x+4)$.

$$a = \underline{\hspace{2cm}} \quad p = \underline{\hspace{2cm}} \quad q = \underline{\hspace{2cm}}$$

- Enter $y = (x-2)(x+4)$ into the Y= screen of your calculator, and use a table to help you graph points in the given coordinate plane. Connect with a smooth curve with arrows on both ends.



- Give the coordinates of the:

vertex:

y-intercept:

x-intercept(s):

- How is the x-coordinate of the vertex related to the x-intercepts?

X-intercepts:

6. Write an equation in intercept form for a parabola with the given x-intercepts. Recall that intercept form is $y = a(x - p)(x - q)$, where p and q are the x-intercepts. Use $a=1$.

a. 2 and 5

b. -2 and -5

c. -3 and 7

$y = (\quad)(\quad)$

$y = (\quad)(\quad)$

$y = (\quad)(\quad)$

7. Find the x-intercepts of these equations:

a. $y = (x - 1)(x - 9)$

b. $y = (x - 6)(x + 2)$

c. $y = (x + 7)(x + 1)$

x-ints: _____ and _____

x-ints: _____ and _____

x-ints: _____ and _____

Vertex:

8. In #5d, you should have noticed that the x-coordinate of the vertex is the average of the x-intercepts. Average each pair of answers in #7 to find the x-coordinate of the vertex.

a. $y = (x - 1)(x - 9)$

b. $y = (x - 6)(x + 2)$

c. $y = (x + 7)(x + 1)$

x of vertex = _____

x of vertex = _____

x of vertex = _____

9. To find the y-coordinate of the vertex, plug your x-value from #8 into its corresponding equation and simplify. Show your work below.

a.

b.

c.

Vertex (,)

Vertex (,)

Vertex (,)

Y-intercept:

10. The y-intercept of any graph always has an x-coordinate of 0. Plug in 0 for x in each equation, and simplify to find the y-intercept.

a. $y = (x - 1)(x - 9)$

b. $y = (x - 6)(x + 2)$

c. $y = (x + 7)(x + 1)$

y-int = _____

y-int = _____

y-int = _____

Graph:

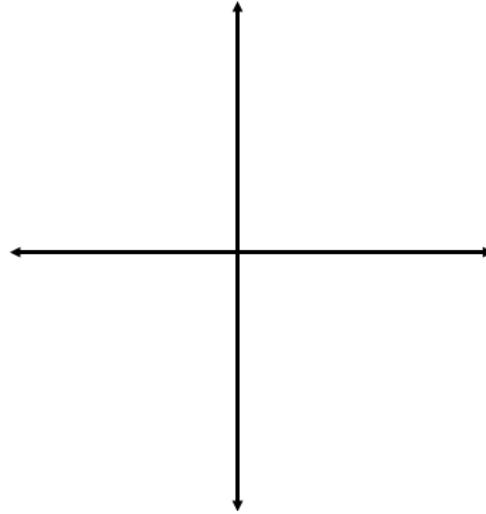
11. Copy your answers from #7, 9, and 10 in the appropriate spaces. Graph the intercepts and vertex, and connect to form a parabola. Label the points with their coordinates.

a. $y = (x-1)(x-9)$

x-ints:

vertex:

y-int:

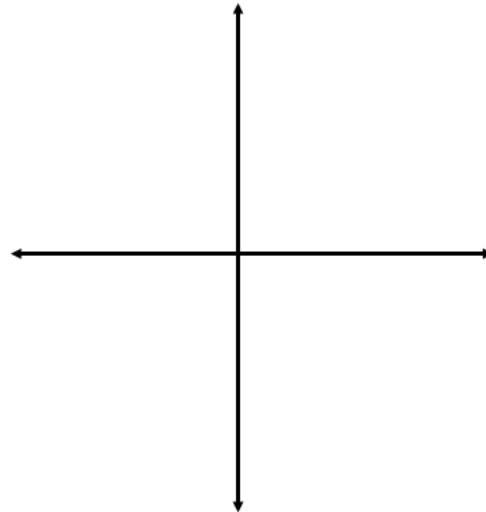


b. $y = (x-6)(x+2)$

x-ints:

vertex:

y-int:

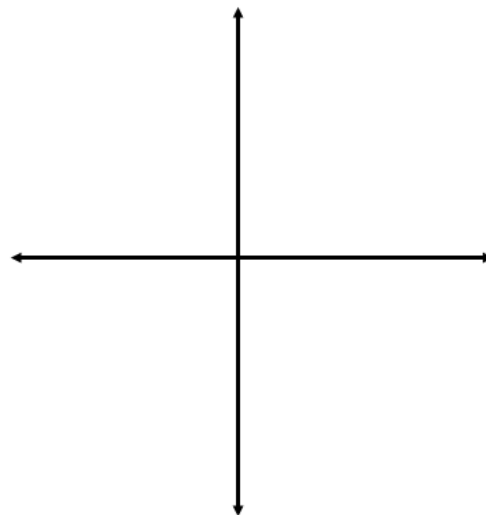


c. $y = (x+7)(x+1)$

x-ints:

vertex:

y-int:



FINDING 'a' FOR EQUATIONS IN INTERCEPT FORM (Section 4.10)

1. Follow the steps to write an equation in intercept form $y = a(x - p)(x - q)$ for the given parabola.

- a. Substitute the x-intercepts for p and q in the equation.

$$y = a(x - \underline{\hspace{2cm}})(x - \underline{\hspace{2cm}})$$

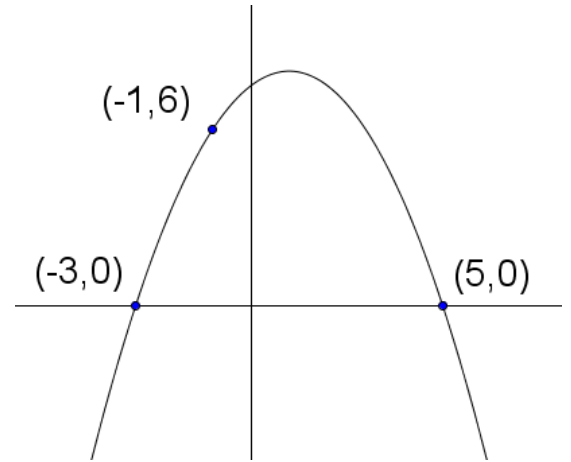
- b. Substitute the coordinates of another point on the parabola for x and y.

$$\underline{\hspace{2cm}} = a(\underline{\hspace{2cm}})(\underline{\hspace{2cm}})$$

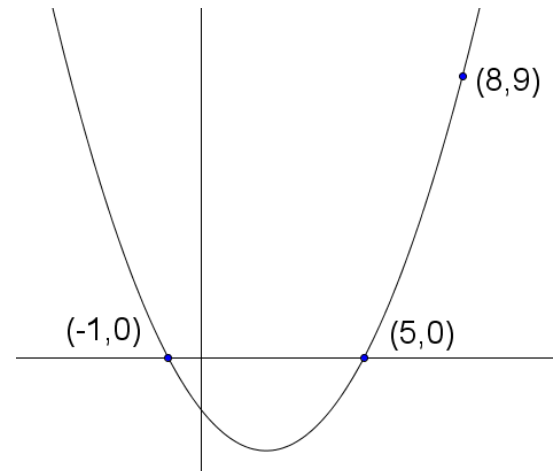
- c. Solve for a. Show your work below.

- d. Rewrite the equation so that x and y remain as variables and all other parts of the equation are replaced by their numeric values.

$$y = \underline{\hspace{2cm}}(x \underline{\hspace{2cm}})(x \underline{\hspace{2cm}})$$



2. Repeat the steps above to write an equation in intercept form for the graph shown at right.



Factoring $x^2 + bx + c$ (Section 4.3)

1. Factor each expression.

a. $x^2 + 8x + 7$

b. $x^2 - 8x + 7$

c. $x^2 - 6x - 7$

d. $x^2 + 16x + 15$

e. $x^2 - 8x + 15$

f. $x^2 + 2x - 15$

g. $x^2 - 25$

h. $x^2 - 49$

i. $x^2 - 9x$

2. We use factoring to convert a quadratic equation from standard form $y = ax^2 + bx + c$ to intercept form $y = a(x - p)(x - q)$. Factor each, and find the x-intercepts. (The value of a is 1; so don't write anything for a.)

a. $y = x^2 - 3x - 18$

b. $y = x^2 + 5x - 14$

c. $y = x^2 - 10x + 9$

Intercept form:

Intercept form:

Intercept form:

x-intercepts:

x-intercepts:

x-intercepts:

NOTE: The x-intercepts of a function are also known as its zeroes because these values cause y to equal 0. For example, the zeroes of $y = x^2 - 3x - 18$ are 6 and -3 because 6 and -3 are the x-intercepts of its graph and plugging in 6 or -3 for x in the equation makes y equal 0.

3. Find the zeroes of each equation:

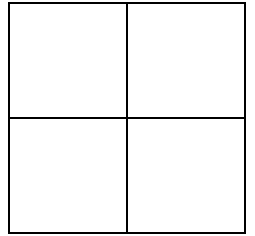
a. $y = x^2 - 36$

b. $y = x^2 - x - 12$

c. $y = x^2 + 2x$

Factoring $ax^2 + bx + c = 0$ When a is Not 1 (Section 4.4)

1. (Us) To factor $2x^2 - 5x + 3$.



a. Write $2x^2$ in the upper left box and 3 in the lower right box.

b. Now that our first term is $2x^2$ instead of x^2 , we split that into $2x$ and x (instead of x and x). Write one of these ($2x$ or x) outside the diagram above the first column of boxes and the other to the left of the first row. This shows that $2x \cdot x = 2x^2$.

c. The number 3 has only one pair of factors: 1 and 3. Write these numbers (1 and 3) outside the diagram above the second column of boxes and the other to the left of the second row.

d. Fill empty boxes by multiplying base times height and writing the area in the appropriate box.

e. Experiment with the signs (positive or negative) of the 1 and 3 outside the diagram until the upper right and lower left boxes add up to $-5x$. If you can't get it to work, switch the positions of the 1 and 3 outside the diagram, and go back to part d.

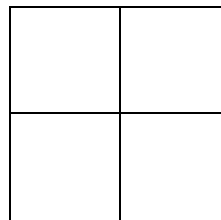
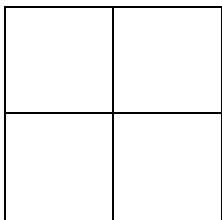
f. When you're done, the 4 boxes inside the diagram should add up to $2x^2 - 5x + 3$, and the outer part should show the factors. Write the factors of $2x^2 - 5x + 3$ below:

()()

2. (You) Now try factoring these:

a. $2x^2 + 7x + 5$

b. $3x^2 - 20x - 7$ (Write -7 in the last box.)



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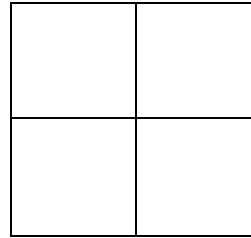
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It may take a little longer to find the correct combination for these. The c-values have more than one pair of factors, and only one pair will work. To limit frustration, keep track of what you've already tried.

3. (Us) Factor $2x^2 + 13x - 15$.

List factor pairs of 15 here:

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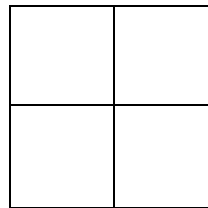
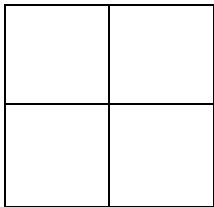
4. (You) Try factoring these:

a. $2x^2 - 3x - 14$

b. $3x^2 - 13x + 12$

List factor pairs of 14 here:

List factor pairs of 12 here:



You won't need a diagram for these. They all fit the pattern shown below, called a **difference of squares**.

DIFFERENCE OF SQUARES

Product: $a^2 - b^2$

Factors: $(a+b)(a-b)$

5. (You) Factor:

a. $16x^2 - 25$

b. $100x^2 - 1$

c. $4x^2 - 81$

Zero Product Property & Factoring with GCF's (Section 4.4)

The Zero Product Property

If a product is zero, then one or more of the factors must be zero.

For example, if $(x+2)(3x-1)=0$, then either $x+2=0$ or $3x-1=0$.

To use the Zero Product Property to solve, set each factor equal to 0, and solve the separate equations for x .

1. $(x+3)(x-6)=0$

_____ = 0 or _____ = 0

Set each factor equal to 0.

Solve the separate equations for x .

$x =$ _____ or $x =$ _____

2. $(2x-10)(4-x)=0$

_____ = 0 or _____ = 0

$x =$ _____ or $x =$ _____

3. $x(x-8)=0$

_____ = 0 or _____ = 0

$x =$ _____ or $x =$ _____

When an expression has a common factor, pull the common factor first.

4. Follow the steps below to factor $6x^2 - 4x - 10$:

a. What is the greatest common factor of $6x^2$, $4x$, and 10? _____

b. Write the GCF outside the parentheses, and in the parentheses, write the expression by which you would multiply to get the original problem back.

_____ ()

c. Keep the GCF outside the parentheses, and factor the expression in parentheses.

_____ () ()

5. Factor by removing the GCF first.

a. $6x^2 + 9x - 27$

b. $-6x^2 + 16x - 8$

REVIEW FOR 4.1 – 4.4 INTERCEPT FORM

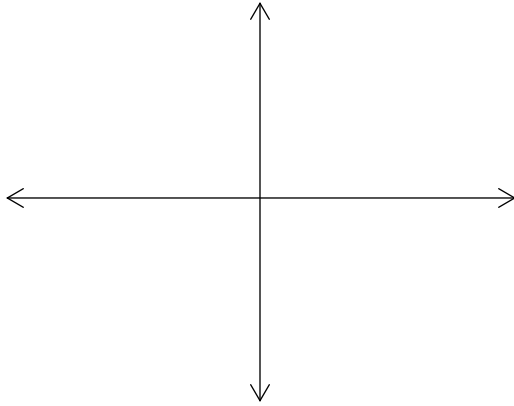
1. Find the x -intercepts, vertex, and y -intercept. Then sketch a graph, and label these points.

a. $y = (x+2)(x-4)$

x -intercepts:

vertex:

y -intercept:

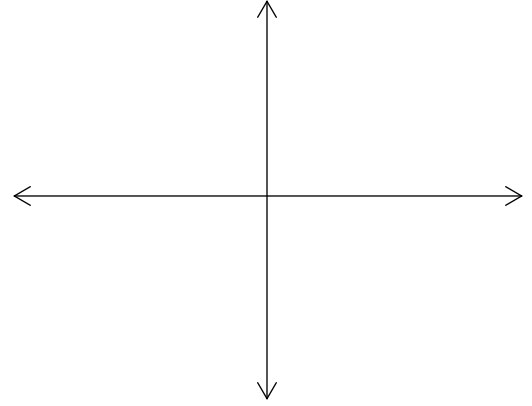


b. $y = 3(x-1)(x-4)$

x -intercepts:

vertex:

y -intercept:

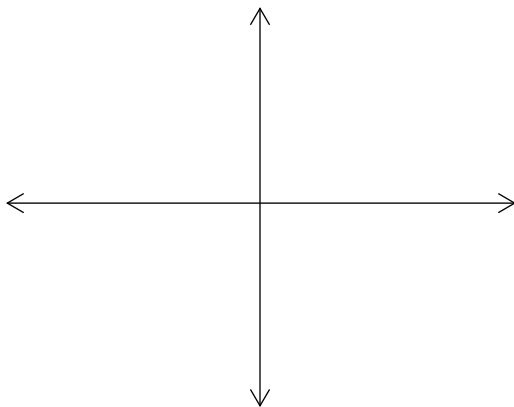


c. $y = -(x+3)(x-1)$

x -intercepts:

vertex:

y -intercept:

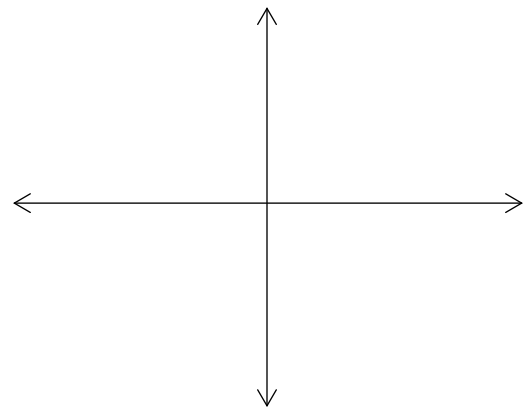


d. $y = 2(x+1)(x-2)$

x -intercepts:

vertex:

y -intercept:



2. Multiply and simplify to convert to standard form.

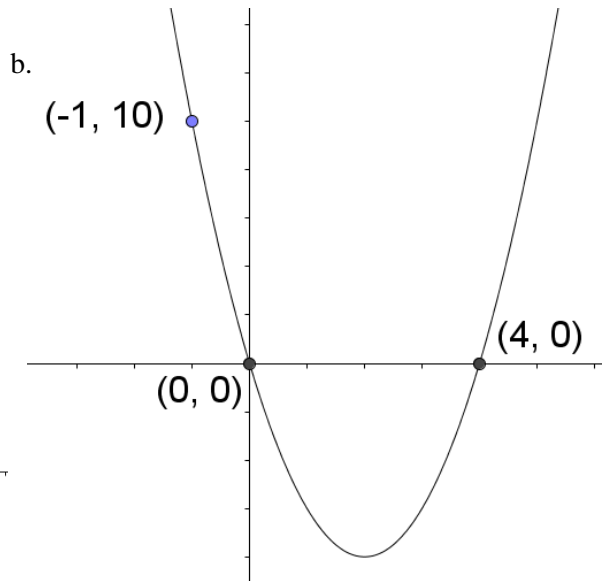
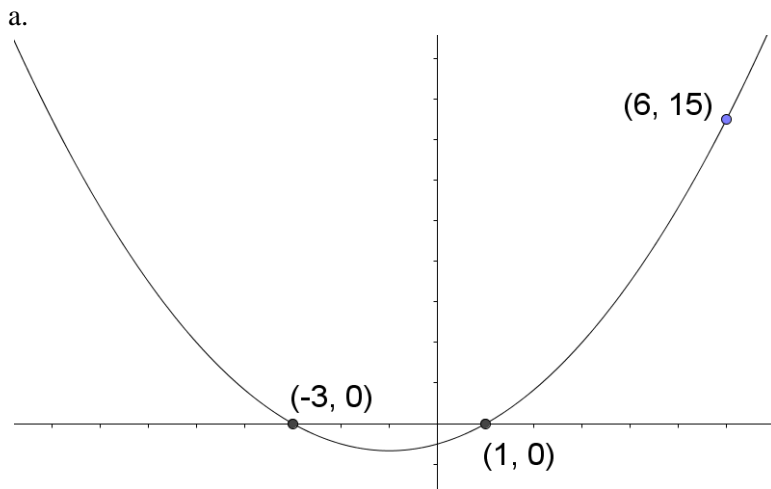
a. $y = (x+2)(x-4)$

b. $y = 3(x-1)(x-4)$

c. $y = -(x+3)(x-1)$

d. $y = 2(x+1)(x-2)$

3. Find an equation in intercept form for each graph:



4. Factor the left side of each equation, and use the Zero Product Property to solve. **Show the work for checking your answers.**

a. $x^2 + x - 6 = 0$

b. $x^2 - 5x + 6 = 0$

c. $x^2 + 7x + 12 = 0$

d. $x^2 - 36 = 0$

e. $x^2 - 11x + 18 = 0$

f. $2x^2 + 3x - 2 = 0$

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4. Factor the left side of each equation, and use the Zero Product Property to solve. **Show the work for checking your answers.**

g. $9x^2 - 4 = 0$

h. $2x^2 - 3x - 9 = 0$

i. $2x^2 + 11x + 15 = 0$

j. $7x^2 - 3x = 0$

k. $3x^2 + 10x - 8 = 0$

l. $3x^2 + 9x - 30 = 0$

5. Factor to write in intercept form. Then find the x-intercepts, vertex, and y-intercept, and graph.

a. $y = x^2 - 5x + 6$

b. $y = 2x^2 - 4x - 6$

x-intercepts:

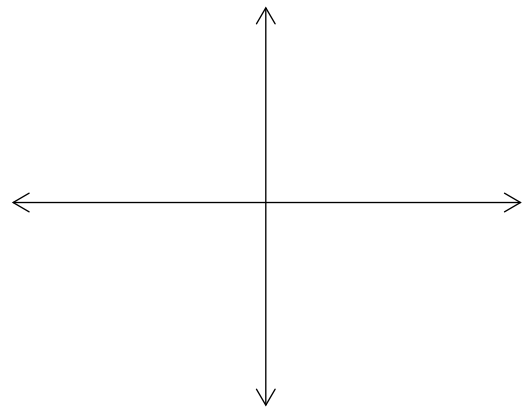
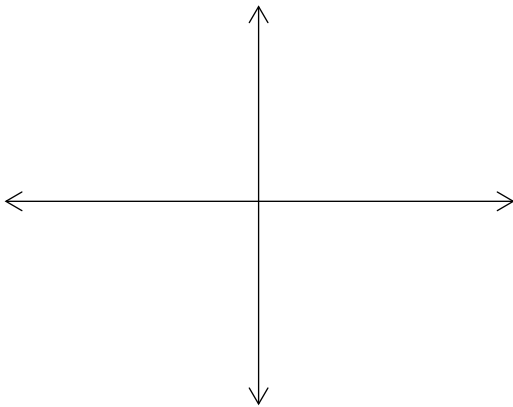
x-intercepts:

vertex:

vertex:

y-intercept:

y-intercept:

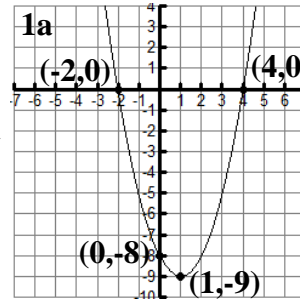


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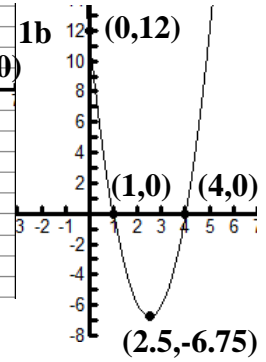
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Answers:

1. a. x-int (-2,0) and (4,0); vertex (1,-9); y-int (0,-8); graph →



b. x-int (1,0) and (4,0); vertex (2.5, -6.75); y-int (0,12); graph →



c. x-int (-3,0) and (1,0); vertex (-1,4); y-int (0,3); graph ↓

d. x-int (-1,0) and (2,0); vertex (.5,-4.5); y-int (0,-4); graph ↘

2. a. $y = x^2 - 2x - 8$ b. $y = 3x^2 - 15x + 12$

c. $y = -x^2 - 2x + 3$ d. $y = 2x^2 - 2x - 4$

3. a. $y = 1/3(x+3)(x-1)$ b. $y = 2x(x-4)$

4. a. $(x+3)(x-2) = 0$; $x = -3$ or $x = 2$

b. $(x-3)(x-2) = 0$; $x = 3$ or $x = 2$

c. $(x+4)(x+3) = 0$; $x = -4$ or $x = -3$

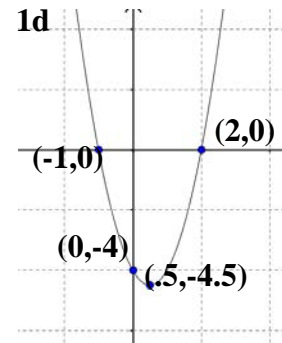
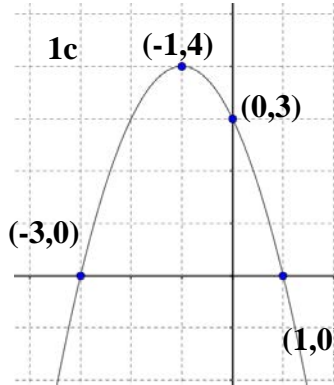
d. $(x+6)(x-6) = 0$; $x = -6$ or $x = 6$

e. $(x-9)(x-2) = 0$; $x = 9$ or $x = 2$

f. $(2x-1)(x+2) = 0$; $x = 1/2$ or $x = -2$

g. $(3x+2)(3x-2) = 0$; $x = -2/3$ or $x = 2/3$

h. $(2x+3)(x-3) = 0$; $x = -3/2$ or $x = 3$



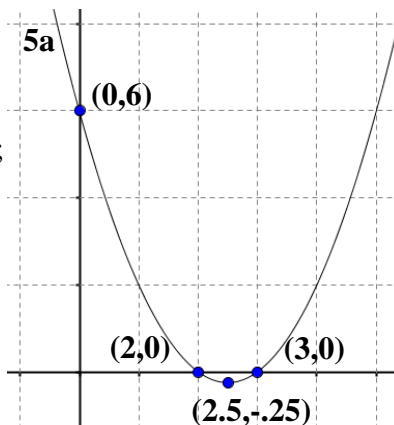
i. $(2x+5)(x+3) = 0$; $x = -5/2$ or $x = -3$

j. $x(7x-3) = 0$; $x = 0$ or $x = 3/7$

k. $(3x-2)(x+4) = 0$; $x = 2/3$ or $x = -4$

l. $3(x+5)(x-2) = 0$; $x = -5$ or $x = 2$

5. a. x-int (2,0) and (3,0); vertex (2.5,-.25); y-int (0,6); graph →



b. x-int (3,0) and (-1,0); vertex (1,-8); y-int (0,-6); graph →

